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CONSTANT TEMPERATURE CREEP STUDIES OF SOME SIMPLE POLYMERIC COMPOSITES

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CONSTANT TEMPERATURE CREEP STUDIES OF SOME SIMPLE POLYMERIC COMPOSITES

by

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ABSTRACT

An investigation of uniaxial creep in simple synthetic composites of polyethylene and polypropylene was made to determine the parametric behavior and interrelation of each component by varying the relative volume and interfacial contact area. A mathematical model was developed and used to predict the experimental behavior which was determined by least squares fitting of the data. A digital computer was used in the analysis and good correlation between the experimental measurements and theoretical predictions was obtained. Included is a report of the design and construction of equipment for theoretically meaningful viscoelastic measurements.

TABLE OF CONTENTS

SECTION			
1.	Introduction	11	
2.	2. The Concept of Composite Structural Materials		
3.	Basic Theory of Linear Viscoelasticity		
4.	4. Statement of the Problem		
5.	5. The Experiment		
6.	. Analysis of the Data		
7.	Conclusions		
BIBLIOGRAPHY			
APPENDICES			
Α.	Tabulated Data	51	
В.	Fortran 60 Computer Programs Used in Analysis	55	
C.	Special Equipment	67	

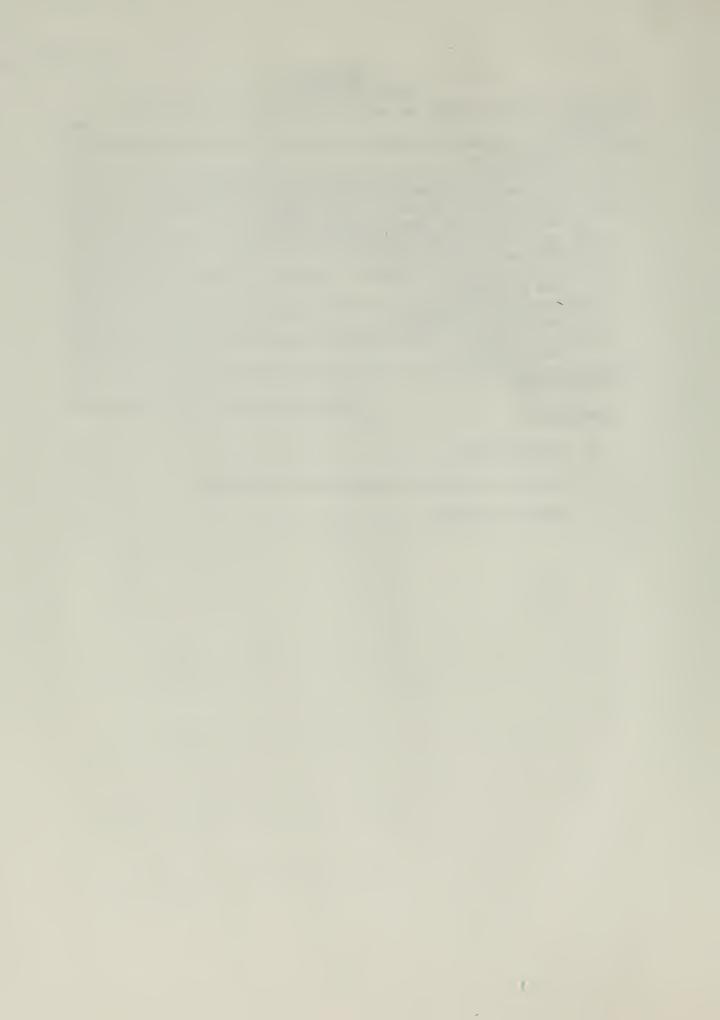
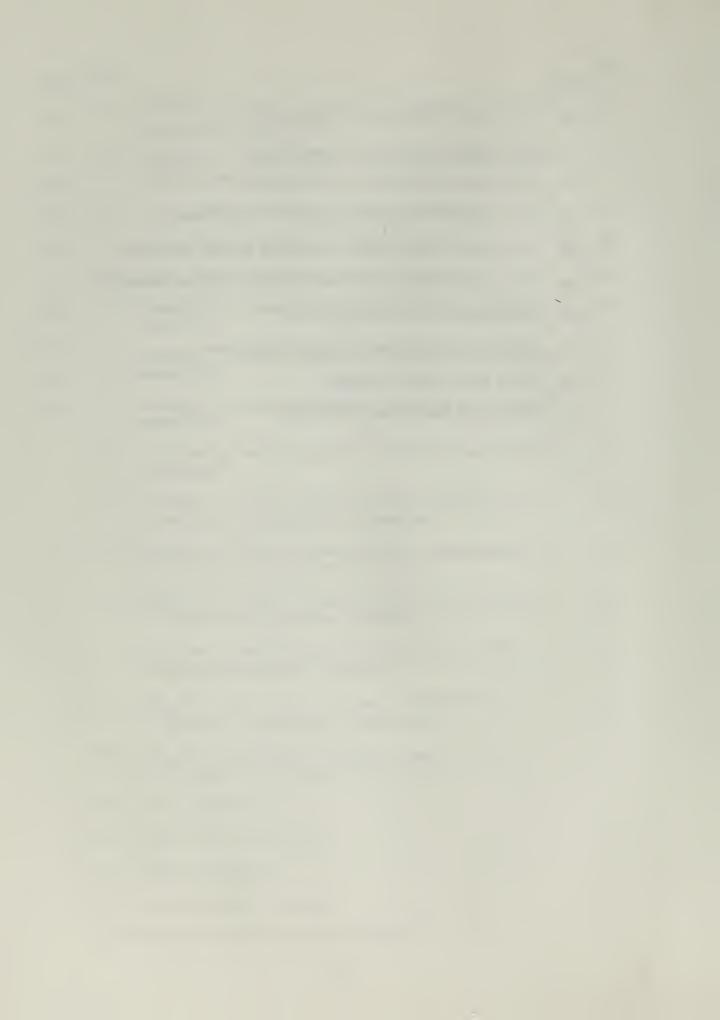


TABLE OF ILLUSTRATIONS

FIGURE		PAG	
1.	The Hookian Solid	16	
2.	The Newtonian Liquid	16	
3.	The Maxwell Model	16	
4.	The Kelvin-Voigt Model	16	
5.	The Standard Linear Solid	17	
6.	Mechanical Behavior of Standard Linear Solid	17	
7.	The Generalized Maxwell Model	17	
8.	The Generalized Kelvin-Voigt Model	18	
9.	Superposition of Time Dependent and Time Independent Elongations in Greep		
10.	Generalized Constant Temperature Creep Curves		
11.	Creep Specimen	25	
12.	Tripolitis Extensometer		
13.	Tripolitis Extensometer Attached to Single Specimen		
14.	Polyethylene/Polypropylene Composite with Clamps	26	
15.	Clamps Used Inside and Outside Gauge	27	
16.	Polyethylene/Polypropylene Composite with 1, 2, 3 & 4 Clamps	27	
17.	Sample CREEPDRW Output	34	
18.	Parameter A versus Stress, Single Components	35	
19.	Parameter A versus Stress, Single Components	35	
20.	Parameter A3 versus Stress, Single Components	36	
21.	Parameter A versus Stress, Single Components	36	
22.	Parameter A versus Stress, 50 % Polyethylene - 50 % Polypropylene Composites	37	
23.	Parameter A, versus Stress, 50 % Polyethylene - 50 % Polypropylene Composites	37	

<u>ר</u>	GURE		PAGI
	24.	Parameter A versus Stress, 50 % Polyethylene - 50 % Polypropylene Composites	38
	25.	Parameter A, versus Stress, 50 % Polyethylene - 50 % Polypropylene Composites	38
	26.	Log A versus Log Stress, Single Components	39
	27.	Log A versus Log Stress, Single Components	40
	28.	Log A ₄ versus Log Stress, Single Components	41
	29.	Parameter A versus Stress for PE/PP/PE and PP/PE/PP Composites	42
	30.	Parameter A versus Stress for PE/PP/PE and PP/PE/PP Composites 2	42
	31.	Parameter A ₃ versus Stress for PE/PP/PE and PP/PE/PP Composites	43
	32.	Parameter A ₁₄ versus Stress for PE/PP/PE and PP/PE/PP Composites	43
	33.	Parameter A versus Volume Fraction Polyethylene in Polyethylene/Polypropylene Composites	ነካተ
	34.	Parameter A versus Volume Fraction Polyethylene in Polyethylene/Polypropylene Composites	71/1
	35.	Parameter A _l versus Volume Fraction Polyethylene in Polyethylene/Polypropylene Composites	45
	36.	E _{ab} versus Volume Fraction Polyethylene in Polyethylene/Polypropylene Composites	45
	37.	(1/K ab) versus Volume Fraction Polyethylene in Polyethylene/Polypropylene Composites	46
	38.	(1/B _{ab}) ¹ versus Volume Fraction Polyethylene in Polyethylene/Polypropylene Composites	46
	39•	Creep Laboratory	70
	40.	Eleven Unit Creep Machine	70
	41.	Lever Arm Assembly	71
	42.	Various Specimen Holders	71
	43.	Five Unit Stress Relaxation Machine	72

FIGURE		PAGI
44.	Clip Gauge Extensometer (Dimensions)	73
45.	Clip Gauge Extensometer (Components)	73
46.	Clip Gauge Extensometers and Brackets	74
47.	Clip Gauge Extensometer Attached to Specimen	74
48.	Clip Gauge Extensometer Wheatstone Bridge Circuitry	75
49.	Clip Gauge Extensometer Theoretical Analysis Schematic	75
50.	Extensometer Calibration Device	76
51.	Clip Gauge Extensometer Calibration Curve	77
52.	Creep Measurement Circuitry	78
53.	Relaxation Measurement Circuitry	78



NOTATION

σ	Nominal stress
$\sigma_{\!\!\scriptscriptstyle o}$	Initial stress
Ġ	Stress rate
ε	Nominal strain
Ė	Strain rate
δ	Elongation
ΔL	Change in length
L	Length
Lo	Initial length
P	Force
A	Cross sectional area
G	Modulus in shear
E	Modulus in tension
η	Coefficient of viscosity
τ	Time constant
t	Time
PE	Polyethylene
PP	Polypropylene

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1. Introduction

The rapid expansion of today's advanced technology is evidenced in the literature by the large number of publications related to the design, fabrication, testing and usage of composite materials. The use of composites as a structural material is not new but the related technology is just now beginning to grow. Almost all of the experimental work on the properties of composites has been performed in direct response to the immediate technological needs. As a result, very little basic research has been reported.

One of the definite voids in the knowledge of the behavior of composite materials is that which is associated with its viscoelastic properties. [5, 7, 14] * This viscoelastic behavior is evidenced in the creep and stress relaxation mechanisms of the composite and needs to be related to the interaction between the basic components of the composite.

The primary purpose of this study was to investigate the constant load creep behavior of a simple synthetic composite of known structure. Measurements were obtained at a temperature of 23*1 degree C and a relative humidity of 50 * 2 % in accordance with ASTM Standards. [15] A comparison of the behavior of the variously constructed composites with that of the parent materials was attempted by least squares fitting of the data using standard digital computer techniques and graphical analysis.

Appendix C is a report of the design and establishment of a Laboratory for the purpose of making these measurements.

^{*}Numbers in brackets refer to Articles in the Bibliography.

$$\varepsilon = \frac{G_0}{E_0} + \sum_{i=1}^{n} \frac{G_0}{E_i} \left[1 - \exp\left(-\frac{t}{\tau_i}\right) \right] + \frac{G_0}{\eta(t)} t$$

Where

$$T_i = \eta i/E_i$$
 $T_0 = initial stress.$

Simplifying the system further by letting n = 1;

$$\varepsilon = \frac{\sigma_0}{E_0} + \frac{\sigma_0}{E_1} \left[1 - \exp\left(-\frac{t}{\tau_1}\right) \right] + \frac{\sigma_0}{\gamma} t.$$

The single element Voigt Model or a suitable modification thereof can easily be used to describe the creep behavior of a material in response to a given initial stress G_0 by superposition of the linear terms in stress and time. Marin [13] has extended these simple relations to provide implicitly for a variety of stress levels and has shown that these relations can be fitted to the actual response of many metallic and polymeric materials. Of various suggestions, the following generalized equation of Marin and Pao [22] seems to have the simplest form and follows the superposition principle quite readily:

 $\varepsilon = \frac{\sigma}{E} + K\sigma^n [1 - \exp(-Pt)] + B\sigma^m t.$

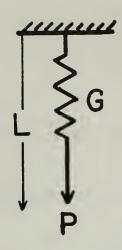
Figure 9 shows how this equation appears using the superposition principle for one given stress level. Figure 10 is a family of creep curves constructed using Marin's equation for five different stress levels. Such a family has been shown to depict the viscoelastic behavior of a simple polymer quite well.

In addition to its application to the structure for a given stress level, we can apply the above principles to the Generalized Maxwell Model of m elements and obtain an equation which describes

the constant strain behavior (stress relaxation) of a material. The following equation may be obtained:

$$\sigma = \varepsilon_o \sum_{i=1}^m E_i \exp(-\frac{t}{\tau_i})$$

A complete solution of these systems is found in reference [10].



P = GL

FIGURE 1
The Hookian Solid

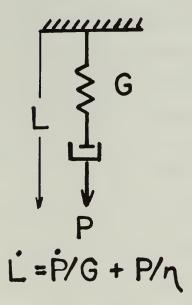


FIGURE 3

The Maxwell Model

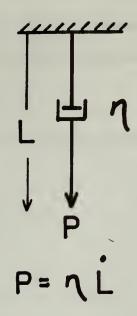


FIGURE 2

The Newtonian Liquid

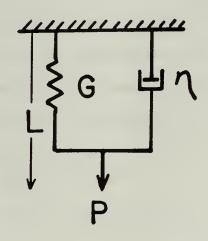
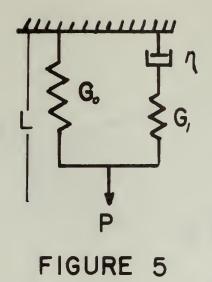
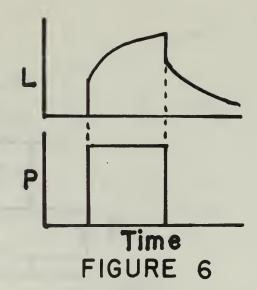


FIGURE 4

The Kelvin-Voigt Model





The Standard Linear Solid

Mechanical Behavior of Standard Linear Solid

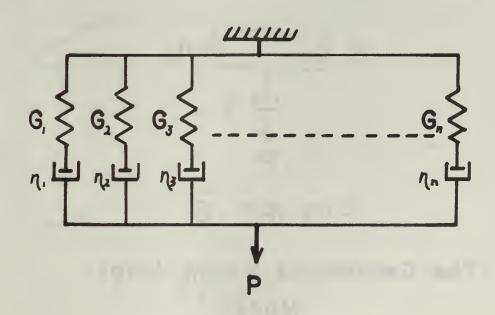


FIGURE 7
The Generalized Maxwell
Model

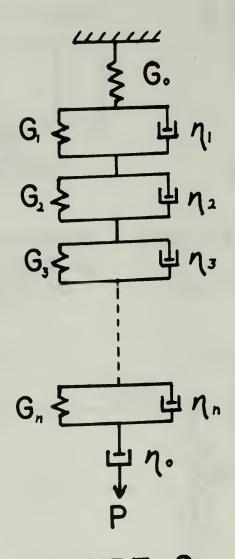


FIGURE 8

The Generalized Kelvin-Voigt

Model

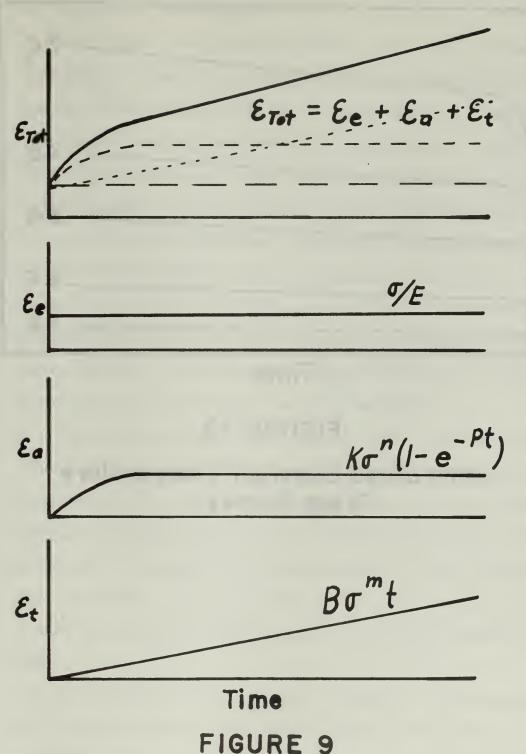


FIGURE 9

Superposition of Time Dependent and Time Independent Elongations in Creep

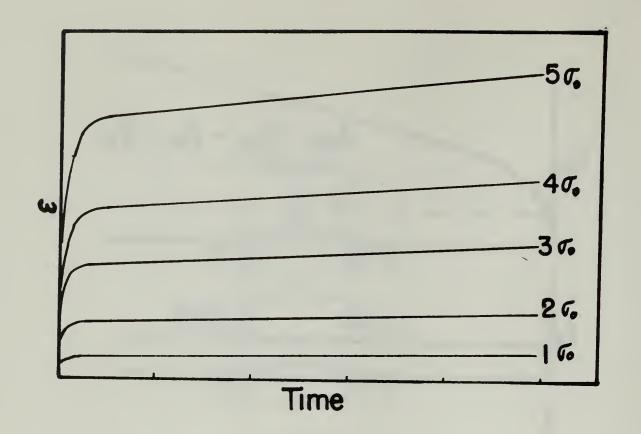


FIGURE 10

Generalized Constant Temperature
Creep Curves

4. Statement of the problem.

The purpose of this study was to investigate the creep behavior of two polymeric materials which were mechanically laminated into a composite structure readily described by simple numbers. A series of constant load machines were used for load application. Linear high density polyethylene and isotactic polypropylene were chosen as the basic materials. Both materials are readily available commercially and both have known composition. Each creeps to a relatively large extent at room temperature using low stresses. Both are crystalline to some degree and their structures may well vary with extension. This property is one which will allow further study of the molecular mechanisms involved. The possible extension of this work is an additional reason for the choice of these particular materials.

The creep measurements were relatively short time tests with an average elapsed time of 100 hours. The variables studied were relative cross-sectional area and the effect of interfacial bonding between the laminates at various stress levels. The data obtained were compiled and fitted to a simplified mathematical model of creep (after Marin) as described above. From this fitted equation, the parameters E, K, n, m, P and B were obtained for each composite and a direct comparison was made.

It was necessary for this study to establish a suitable laboratory for the measurement of creep. This required the installation of temperature and humidity control equipment of sufficient size to maintain the temperature and relative humidity throughout the laboratory. Also, since the obtaining of data is a tedius and time consuming process, a method of recording the data for many units was devised. This equipment is described in Appendix C.

5. The Experiment.

Specimens of linear high density polyethylene and isotactic polypropylene were made with the dimensions of Figure 11. The extruded sheet material was cut into random rectangles oriented parallel, perpendicular and diagonally to the extrusion direction. These specimens were marked according to location and orientation in the sheet and used in groups for creep measurements. Preliminary experimental work used the phosphor bronze clip extensometers described in Appendix C, but the data reported here was obtained using Tripolitis type extensometers. (See Figs. 12 and 13.) The gauge length for all specimens was two inches and the extensometer could be accurately read to + 0.0002 inches. Readings of the data were taken at 2, 5, 10, 15, 20, 25, and 30 seconds total elapsed time, thence varying intervals from 10 seconds to 5 minutes in the first 15 minutes, 5 minutes to 15 minutes in the next 30 minutes and 15 minutes to 30 minutes to approximately 4 hours. Various additional readings were taken to sufficiently determine the extension over a range of 90 to 150 hours total elapsed time. An average of 40 data points were taken on the low stress runs and 60 data points on the high stress runs. An example of the data is enclosed in the test program found in Appendix B.

The temperature was maintained at 23 * 0.8 degrees C throughout the experiment. It is noted that this degree of temperature control was barely adequate for these materials at low stress levels due to the high coefficient of thermal expansion. The relative humidity was not critical due to the nature of the materials but was maintained within the range 48 to 60 %.

Constant load Creep machines (See Appendix C.) with calibrated lever type loading arms and either universal joints or ball and socket

joints above and below the specimen to eliminate any possible torsion or bending of the specimen were utilized.

Initially, many measurements of the creep of single material polyethylene or polypropylene were obtained. The statistical spread of the data was determined to be due largely to the inhomogeneity of the material, and these errors such as measurement errors, errors due to temperature and relative humidity fluctuations and roundoff errors in compiling and processing the data were masked by the statistical spread.

The next series of tests were carried out on two single layers of polyethylene and polypropylene clamped together in parallel with 80 mesh abrasive uniformly spread throughout the interfacial area. These clamps, shown in Figure 14, were placed outside the gauge length to insure the uniform extension of the material between the clamps. Periodic tightening of these outside clamps prevented any relative motion or slippage between the layers. Further tests in this series were made using two polyethylene and one polypropylene or two polypropylene and one polyethylene layers clamped in parallel as in the previous case.

The second series of tests was run on singly layered polyethylene and polypropylene composites using the previously mentioned outside clamps and auxiliary clamps within the gauge length (See Figs. 14 and 15.) to ensure that contact existed between the materials used. The auxiliary clamps within the gauge length were attached in such a way that only a line of contact was made between the clamp and the specimen, thereby eliminating any restraint on the individual fibers of the material except at the line of contact. The imbedded 80 mesh

abrasive served to increase the interfacial contact at points in the vicinity of the clamps by a pinning action. A series of 1, 2, 3, and 4 of these auxiliary clamps were used on the two layer composites at different stress levels to determine the effect of an interface and any shear stresses between the laminates. (See Fig. 16.) All runs were made at initial stresses of 500, 750, 1000, 1250, 1500 and 1750 psi. These particular stress levels were chosen since polyethylene enters the tertiary creep region above 1750 psi. Polyethylene does not enter this region until above 3000 psi but no correlation of the creep due to the polyethylene component is possible at stresses of this magnitude.

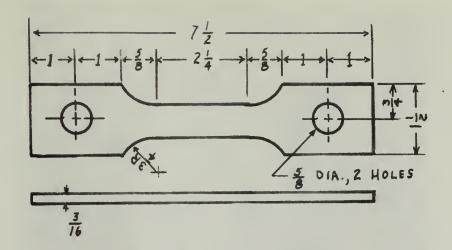


FIGURE II CREEP SPECIMEN

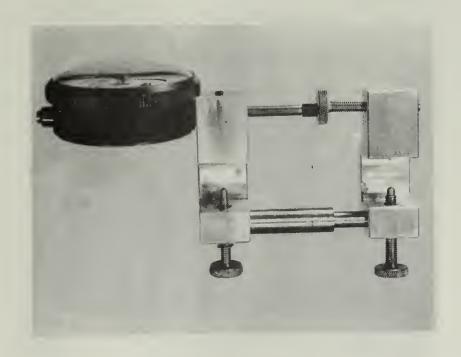
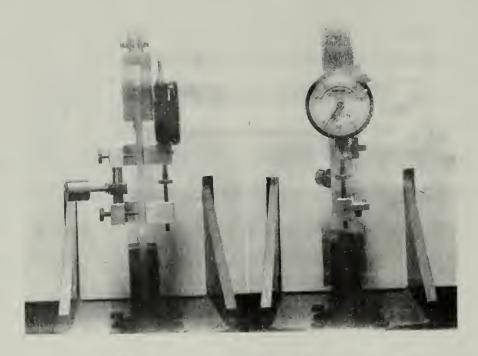


FIGURE 12 TRIPOLITIS EXTENSOMETER



FIGUR TRIPOLITIS EXTENSOMETER

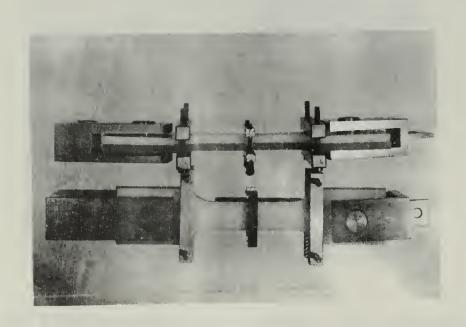


FIGURE 14 PE/PP COMPOSITE WITH CLAMPS

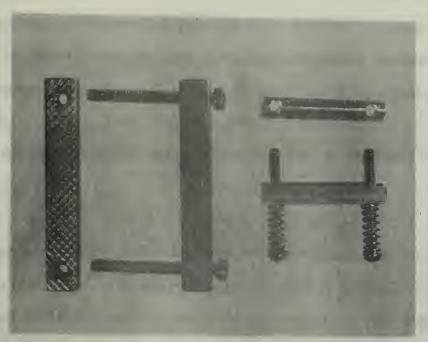


FIGURE 15 CLAMPS USED INSIDE & OUTSIDE GAUGE

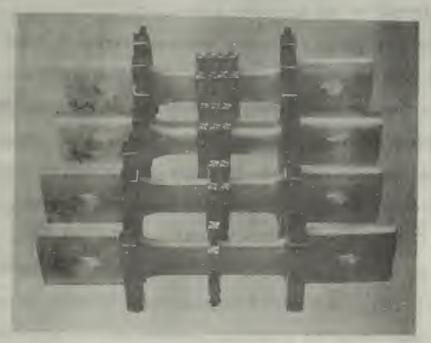


FIGURE 16 PE/PP COMPOSITE WITH 1,2,384 AUX. CLAMPS

6. Analysis of the data.

Creep data in the form of elongation ΔL versus time were obtained for all composite specimens studied. The data were punched on cards and placed in the digital computer via the first program in Appendix B. The output of this program included the input data, the calculated stress, the fraction of each component of the specimen, and the parameters A_i (i = 1, 2, 3, 4) of the equation:

$$\Delta L = A_1 + A_2(1 - \exp(-A_3t)) + A_1t$$

for which the parameters A; correspond with those of Marin's equation:

$$\mathcal{E} = \frac{1}{L_0} \left(\frac{\sigma}{E} + K \sigma^n (1 - \exp(-Pt)) + B \sigma^m t \right).$$

Also included was the difference or error between the fitted equation and the input data points and an estimate of the error in the parameters A_i . Appendix A contains a tabulated compilation of the parameters A_i for all of the specimens investigated.

The second computer program in Appendix B served to compare the input data with the fitted parameters. It was also a very good method of checking the accuracy of the punched data since any gross errors could easily be picked up. Figure 17 is a sample of the output. All data points cannot be shown here since the program used to draw the graph is limited to 30 data points. The continuous curves are drawn using the fitted parameters found by least squaring the data. Although an exact fit is not obtained, the fit is close enough to obtain sufficient information to make a comparison of the effect of relative cross-sectional areas and interfacial contact between the laminates.

Plots of A_1 through A_4 versus stress for single unlayered polyethylene and polypropylene specimens are seen in Figures 18 through 21. Note the statistical spread of data, especially for A_3 versus stress.

Figures 22 through 25 serve as a comparison of the parameters A_i for a simple composite of 50 % polyethylene and 50 % polypropylene. The only clamped areas were outside the gauge length. In addition to these points for the composites, curves for 100 % polyethylene and 100 % polypropylene are included.

From Figure 24 it is apparent that the elastic term of Marin's equation ($\sigma/E = A_1L_0$) is not linear in stress. Figure 26 is a log- \log plot of A_1 versus stress for simple polyethylene and simple polypropylene specimens. The slope is 1.3, indicating that the elastic deformation is in fact dependent on stress to the 1.3 power. Many other materials possess similar nonlinearity and this result is not surprising. Similiarly, a log-log plot of A_2 and A_h versus stress are seen in Figures 27 and 28 respectively. The slope of the graphs are 2.0 and corresponds to the value of n and m in the equalities ($A_2 = (1/L_0)$) $(K\sigma^n)$) and $(A_n = (1/L_o) (B\sigma^m))$. These same values of n and m were also obtained from the data on the composite structures. The term A_3 , which corresponds to a relaxation time constant, is apparently independent of stress. However, the time scale needed for creep measurements makes it difficult to determine such relaxation time constants accurately. The magnitude of the fitted parameter was found to be 1.5 hr. 1 for polyethylene and 0.8 hr. -1 for polypropylene.

Figures 29 through 32 are similar graphs of the parameters A versus stress for triply layered composite specimens. Those composites labeled PE/PP/PE were 67 % polyethylene and 33 % polypropylene and those labeled PP/PE/PP were 67 % polypropylene and 33 % polyethylene. Note the effect of increasing the relative volume (cross-sectional area) of one component relative to the other component. A appears linear with volume for all

stress levels but A_2 and A_h are nonlinear.

This nonlinear response is consistent with the mathematical model of the composite. Assuming that Marin's equation:

$$\varepsilon = \frac{\sigma}{E} + K\sigma^{2}[1 - \exp(-Pt)] + B\sigma^{2}t$$

adequately describes the behavior of these composites and that each component carrys its share of the load dependent only on its fractional cross-section, then the following analysis can be used.

Letting a and b denote the relative fraction of component A and B and using subscripts a, b, and ab to identify component A, B, or the composite AB, an appropriate stress analysis of the composite yields Equations 1, 2 and 3.

$$(1)$$
 a + b = 1

(2)
$$\delta_a = \delta_b = \delta_{ab}$$
 or $\xi_a = \xi_b = \xi_{ab}$

(3)
$$a\sigma_a + b\sigma_b = \sigma_{ab}$$

for time t = 0 in which only the elastic component has a value,

$$\varepsilon_{ab} = \frac{\sigma_{ab}}{\varepsilon_{ab}} = \frac{\sigma_{a}}{\varepsilon_{a}} = \frac{\sigma_{b}}{\varepsilon_{b}}$$

$$\sigma_{a} = \left(\frac{E_{a}}{E_{ab}}\right) \sigma_{ab}$$
 and $\sigma_{b} = \left(\frac{E_{b}}{E_{ab}}\right) \sigma_{ab}$

Combining these equations with Equation 3, one gets Equation 4.

$$(4)$$
 $aE_a + bE_b = E_{ab}$

Equation 4 indicates that the elastic deformation for a composite (term E of the viscoelastic equation) should follow a simple addition law. Figure 36 is a plot of observed values for this term versus fraction of polyethylene in the composite, and shows substantial agreement between this theoretical deduction and observed experimental values.

Similarly, for coefficients K and B of the viscoelastic equation, at time t= $5/A_3$ the term (1 - exp(-Pt)) is substantially unity, and the term B σ^2 t is small. Then

$$\mathcal{E}_{a} = \frac{\sigma_{a}}{E_{a}} + K_{a}\sigma_{a}^{2}$$

$$\mathcal{E}_{b} = \frac{\sigma_{b}}{E_{b}} + K_{b}\sigma_{b}^{2}$$

$$\mathcal{E}_{ab} = \frac{\sigma_{ab}}{E_{ab}} + K_{ab}\sigma_{ab}^{2}.$$

Using Equations 1 and 2 again, one gets

$$K_{a} \sigma_{a}^{2} = K_{ab} \sigma_{ab}^{2} + \left[\frac{\sigma_{ab}}{E_{ab}} - \frac{\sigma_{a}}{E_{a}} \right]$$

$$K_{b} \sigma_{ab}^{2} = K_{ab} \sigma_{ab}^{2} + \left[\frac{\sigma_{ab}}{E_{ab}} - \frac{\sigma_{b}}{E_{ab}} \right]$$

from which

$$\sigma_a = \left(\frac{K}{ab}\right)^{\frac{1}{2}} \sigma_{ab}$$
 and $\sigma_b = \left(\frac{K}{ab}\right)^{\frac{1}{2}} \sigma_{ab}$.

Again applying Equation 3, one obtains

$$a \left(\frac{K_{ab}}{K_{a}}\right)^{\frac{1}{2}} \sigma_{ab} + b \left(\frac{K_{ab}}{K_{b}}\right)^{\frac{1}{2}} \sigma_{ab} = \left(\frac{K_{ab}}{K_{ab}}\right)^{\frac{1}{2}} \sigma_{ab}$$

which is simplified into Equation 5.

(5)
$$a\left(\frac{1}{K_a}\right)^{\frac{1}{2}} + b\left(\frac{1}{K_b}\right)^{\frac{1}{2}} = \left(\frac{1}{K_{ab}}\right)^{\frac{1}{2}}$$

Equation 5 indicates that the reciprocal of the square root of coefficient K for the composite should be an addition of volume fraction of its components. Figure 37 is a plot of this item versus volume fraction polyethylene. It verifies this theoretical deduction.

A similar analysis for coefficient B of the viscoelastic equation indicates that for time much greater than $5/A_3$, Equation 6 can be obtained.

(6)
$$a\left(\frac{1}{B_a}\right)^{\frac{1}{2}} + b\left(\frac{1}{B_b}\right)^{\frac{1}{2}} = \left(\frac{1}{B_{ab}}\right)^{\frac{1}{2}}$$

This deduction is verified by experimental values in Figure 38. The excellent agreement between experiment and deductions based on the viscoelastic equation indicate that these composites do behave as the mathematical model predicts.

The relaxation time parameter A (P of the equation) is not so easily checked. However, similar mathematical manipulations starting with the relation ε_a + ε_b = $2\varepsilon_{ab}$ will yield for finite time t;

$$(1 - \exp(-P_a t)) + (1 - \exp(-P_b t)) = 2(1 - \exp(-P_{ab} t)).$$

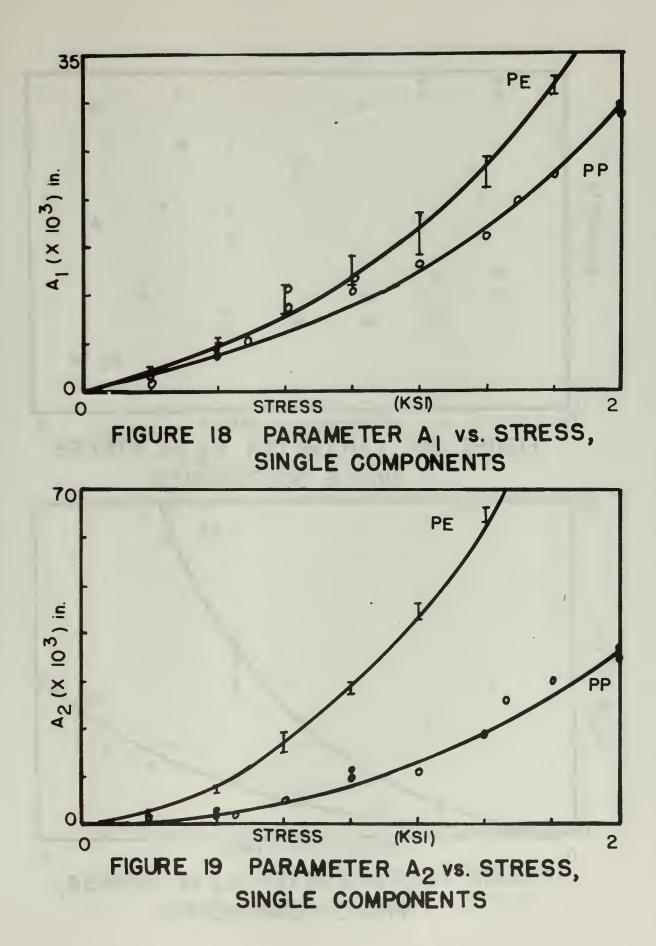
Expanding the exponential terms in a Taylor's Series and truncating the series after the second term, Equation 7 is obtained.

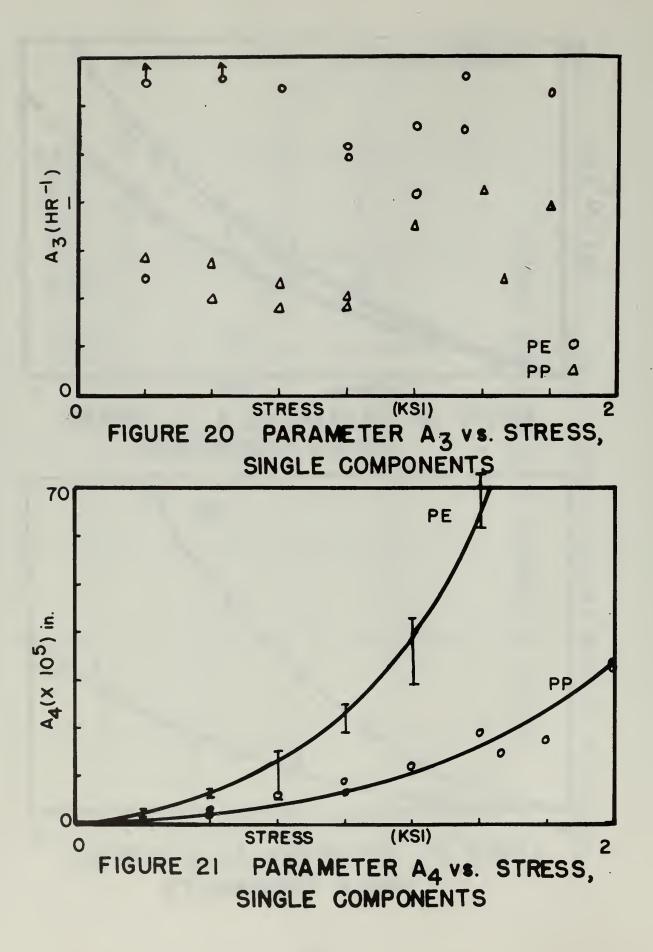
(7)
$$P_{ab} = \frac{P_a + P_b}{2}$$

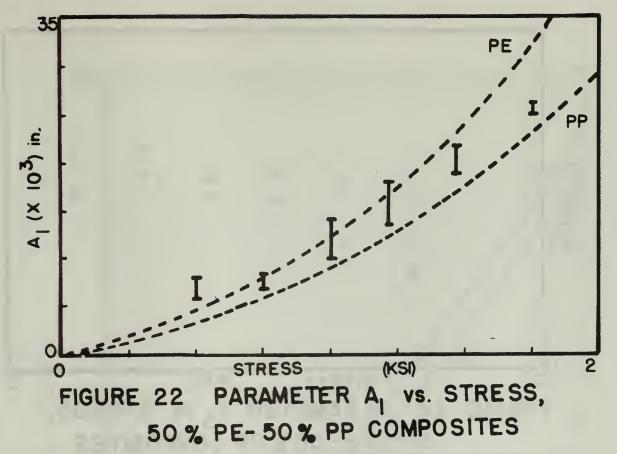
Therefore, the composite's relaxation time constant is the average of the time constants of each component. The scatter of data pertaining to this relaxation time constant precludes any quantitative check of this result. However, most data points do fall within those of 100% polyethylene and 100% polypropylene as predicted. (See Figures 20, 24 and 31.)

The results of increasing interfacial contact points by placing the auxiliary clamps within the gauge length and using 80 mesh carborundum as a keying material showed no apparent effect in the creep behavior of the 50 % polyethylene - 50 % polypropylene composites. The statistical spread of data does not appear to be affected in any way as may be ascertained from comparing the data in Appendix A. The effect of an increased number of specimens at 1000 and 1500 psi. merely increased the spread of the data without significantly skewing it in any specific direction.

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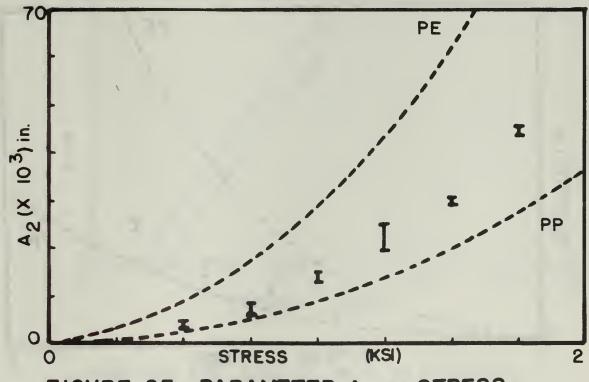


FIGURE 23 PARAMETER A₂ vs. STRESS, 50% PE-50% PP COMPOSITES

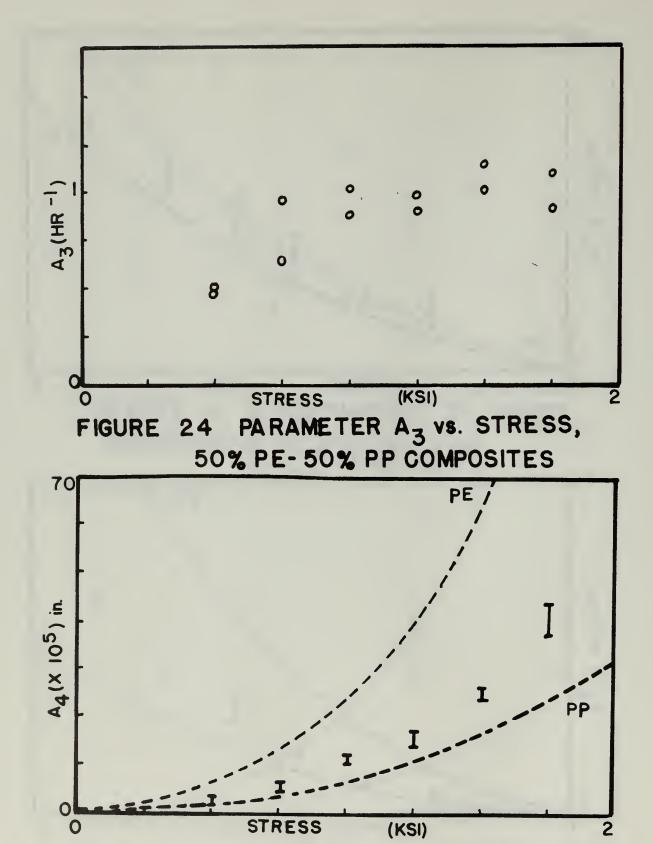


FIGURE 25 PARAMETER A4 vs. STRESS, 50% PE-50% PP COMPOSITES

STRESS

(KSI)

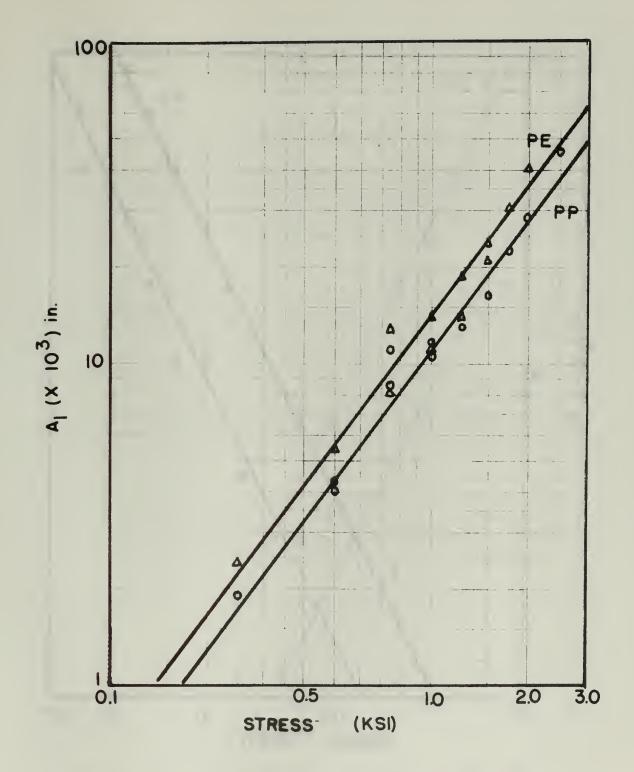


FIGURE 26 Log A₁ vs. Log STRESS,
SINGLE COMPONENTS

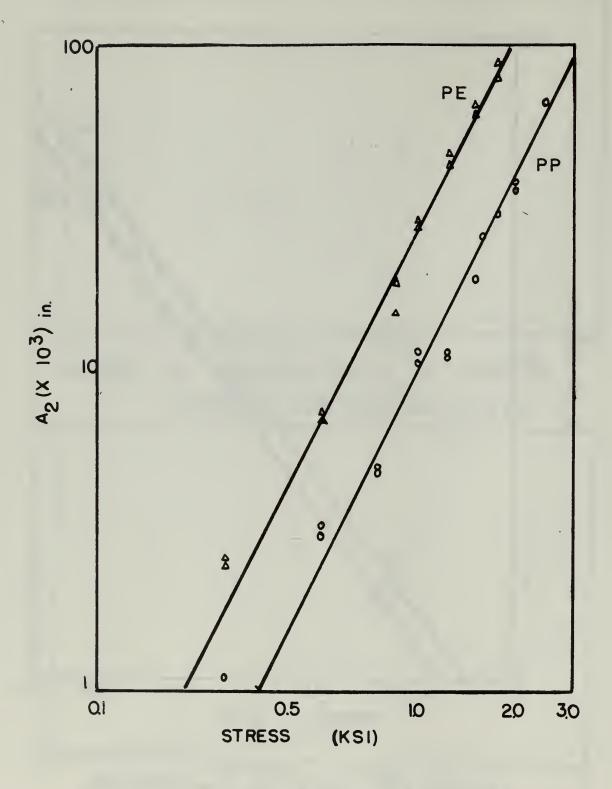


FIGURE 27 Log A₂ vs. Log STRESS, SINGLE COMPONENTS

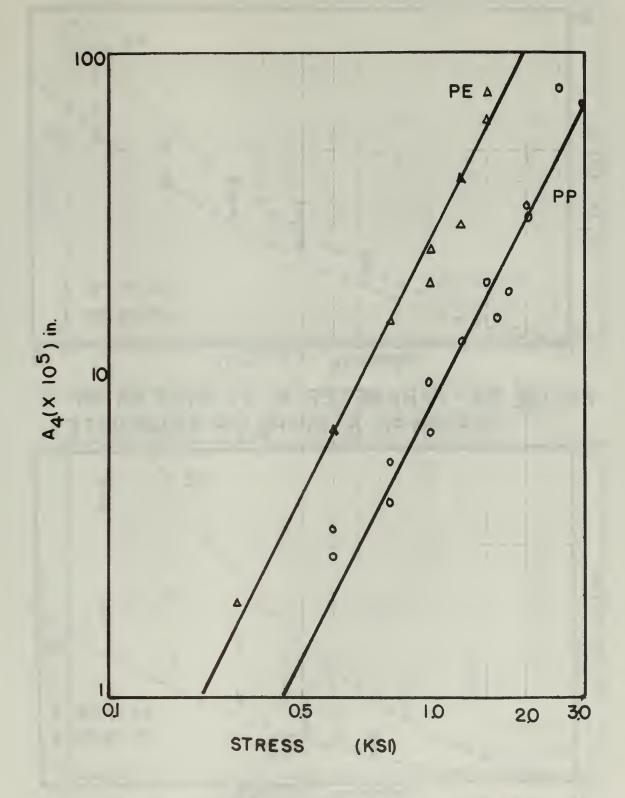


FIGURE 28 Log A4 vs. Log STRESS, SINGLE COMPONENTS

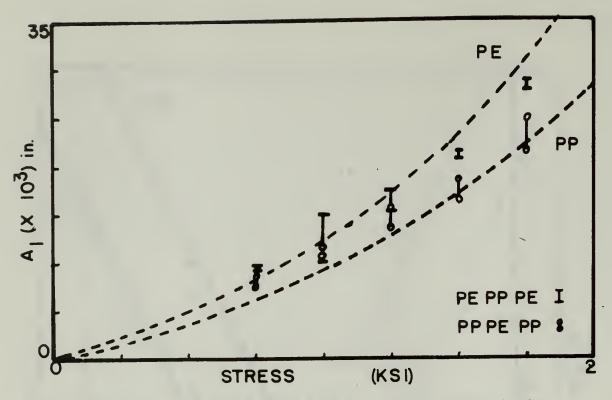


FIGURE 29 PARAMETER A vs. STRESS for PE/PP/PE & PP/PE/PP COMPOSITES

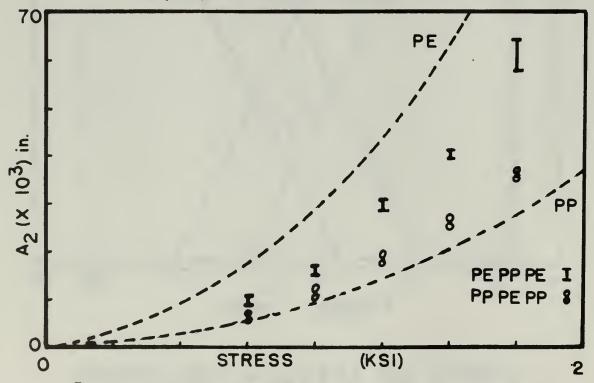


FIGURE 30 PARAMETER A2 VS. STRESS for PE/PP/PE & PP/PE/PP COMPOSITES

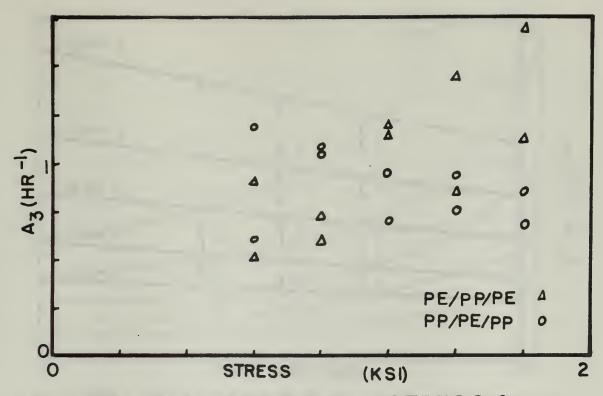


FIGURE 31 PARAMETER A3 vs. STRESS for PE/PP/PE & PP/PE/PP COMPOSITES

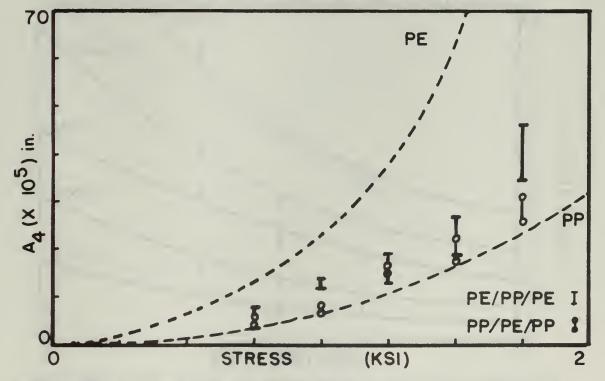


FIGURE 32 PARAMETER A4VS. STRESS for PE/PP/PE & PP/PE/PP COMPOSITES

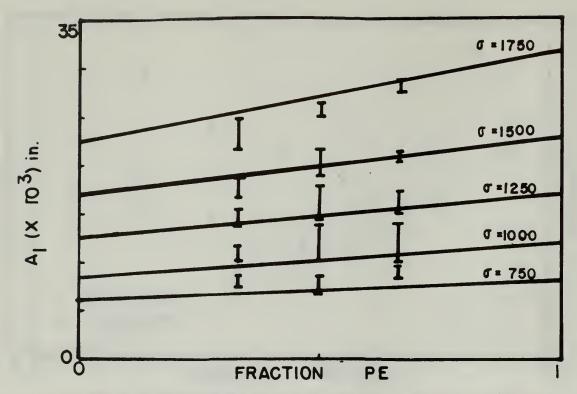


FIGURE 33 PARAMETER A, vs. VOLUME FRACTION PE in PE/PP COMPOSITES

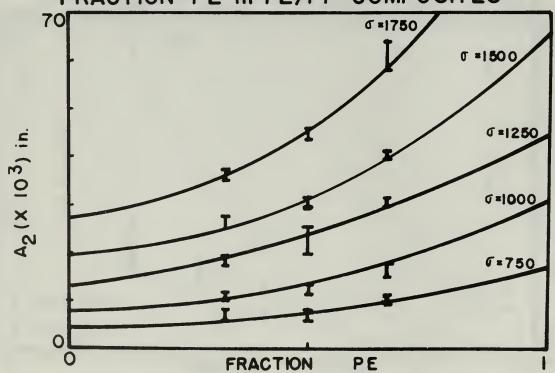


FIGURE 34 PARAMETER A2 VS. VOLUME FRACTION PE in PE/PP COMPOSITES

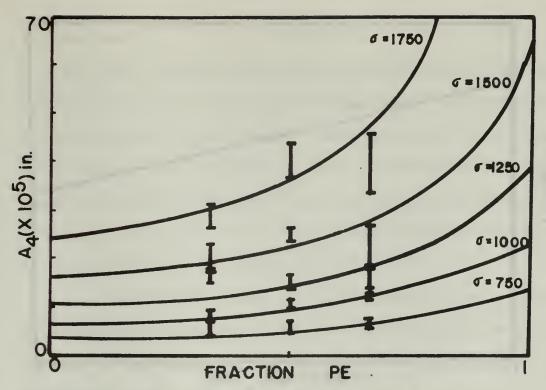


FIGURE 35 PARAMETER A4 vs. VOLUME FRACTION PE in PE/PP COMPOSITES

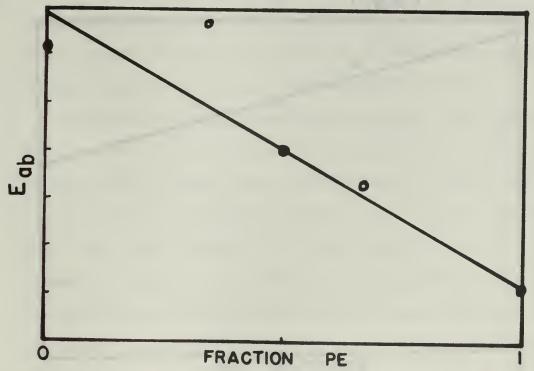


FIGURE 36 Eab vs. VOLUME FRACTION PE in PE/PP COMPOSITES

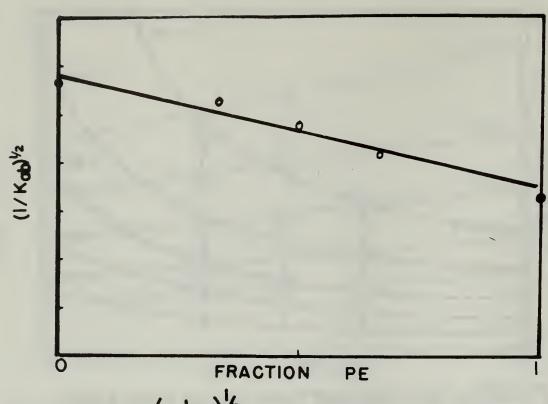


FIGURE 37 $\left(\frac{1}{K_{ab}}\right)^{1/2}$ vs VOLUME FRACTION PE

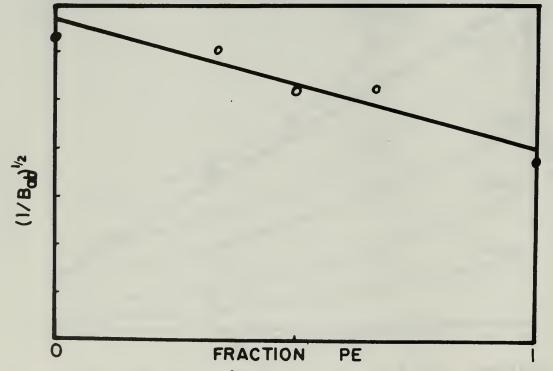


FIGURE 38 $\left(\frac{1}{B_{ab}}\right)^{1/2}$ vs. VOLUME FRACTION PE

7. Conclusions.

The simple laminated composites of polyethylene and polypropylene were prepared and the creep behavior was measured. The parametric characteristics were determined by least squaring the data. These characteristics were compared with those predicted by theoretical analysis and were found to agree quite closely with the predicted results, ie., the creep behavior was a linear function of the properties of each component and its relative volume.

This investigation provides conclusive evidence that a combined mathematical and experimental approach in determining the relative creep behavior of composites is a powerful tool. The basic analysis of the behavior of simple composites is a useful method of predicting creep behavior in the more sophisticated composite systems in addition to the conventional methods for ad hoc measurements. It is necessary, however, that a computer facility be available to the experimenter and that an empirical equation can be used to represent this behavior.

The measurements which were obtained required total elapsed times which were too short for engineering purposes, but any further decrease in creep rate for longer times would have been due largely to the crystallization or strengthening effect as the material elongated under load. This effect was ignored as a first approximation even though it becomes quite apparent in the data. It is to be noted, however, that some polyethylene specimens broke outside the gauge length after one and one-half months under moderate stress. This failure appeared brittle in nature and occurred at the hole in the specimen. These failures would have prevented taking data for times greater than 1000 hours.

The following recommendations are submitted for further work:

- (1) A higher order empirical equation be tested whereby two relaxation time constants can be determined and an allowance for the crystallization or structure factor can be made.
- (2) Stress relaxation data be correlated with simple tension creep data to determine the interrelation through the relaxation mechanisms of the composites.
- (3) Other materials be tested which can be fabricated into true composites of known composition without the difficulties of structural change and poor adhesive properties.

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APPENDIX A
TABULATED DATA BY GROUPS

		S	INGLE F	OIYETHYLENE	SPECIMENS		
RUN	STRESS	% PP	% PE	A ₁ (X 10 ³)	A ₂ (X 10 ³	A ₃ (HR1)	A ₎ (X 10 ⁵)
	(PSI)			(IN.)	(IN.)		(IN./HR.)
1 2 3 4 5 6 7 8	244.4 497.5 742.1 754.0 998.9 1001.6 1244.7 1248.9	0.0	100.0	2.39 5.49 12.91 8.15 14.03 11.27 13.78 18.53	2.68 7.39 19.29 15.40 28.80 27.41 46.49 43.01	0.62 3.78 2.00 1.59 1.24 1.28 1.39	1.93 5.92 4.86 14.80 19.52 24.77 42.67 28.86
9 10 11 12 13	1240.9 1494.5 1502.3 1744.2 1752.9 1995.1			20.89 24.22 30.71 32.54 40.92	63.19 66.18 78.34 88.36 133.60	1.65 1.37 1.55 0.97 0.79	75.12 61.55 294.79 301.00 225.84

		S.	INGLE PO	LYPROPYLENE	SPECIMENS		
RUN	STRESS (PSI)	% PP	% PE	A _l (X 10 ³) (IN.)	A ₂ (X 10 ³) (IN.)	A ₃ (HR1)	A ₄ (X 10 ⁵) (IN./HR.)
1 2 3 4 5 6 7 8 9 10 11 2 13 14 15	251.4 504.3 504.1 613.1 749.1 752.9 999.6 1001.0 1251.6 1501.4 1603.4 1753.6 2003.8 1997.2 2502.3	100.0	0.0	1.91 4.11 4.37 5.52 11.28 8.55 11.71 10.82 12.90 16.35 19.81 22.91 29.08 28.78 45.74	1.10 3.04 3.33 3.04 4.81 4.88 10.45 11.77 11.08 19.02 26.36 30.70 37.75 35.97 68.33	0.73 0.69 0.50 3.52 0.45 0.57 0.46 0.52 0.89 1.06 0.60 0.98 0.88 1.01 0.58	2.20 2.71 3.36 9.96 4.03 5.28 9.45 6.69 12.89 19.47 14.82 18.39 31.44 33.96 77.82

PO	LY ETHYLENI	E/POLYPI	ROPYLENE	COMPOSITES	WITHOUT AUX	ILIARY CLAMPS	
RUN	STRESS	% PP	% PE	A ₁ (X 10 ³)	A ₂ (X 10 ³)	A ₃ (HR. ⁻¹)	A ₄ (X 10 ⁵)
	(PSI)			(IN.)	(IN.)		(IN./HR.)
1 2 3 4 5 6 7 8 9 10	500.4 501.2 748.7 752.0 999.2 1000.0 1252.4 1252.9 1500.0 1481.7	50.7 50.9 50.9 51.4 52.1 51.7 51.5 51.5 51.6	49.3 49.1 49.1 48.6 47.9 48.9 49.3 48.7 48.5 48.9	5.55 8.08 8.70 7.20 10.13 14.14 18.07 13.97 21.57 19.16 26.33	3.62 3.83 6.45 8.56 14.60 13.93 21.06 25.19 30.92 29.99 44.52	2.78 0.48 0.66 0.95 0.88 1.01 0.97 0.90 1.01 1.15	3.53 2.11 5.86 7.20 11.21 11.58 14.31 17.44 26.20 24.16 44.25
11 12	1747.6 1753.1	51.0	48.4 49.0	26.97	44.10	0.94	37.54
	-1771-					-	
PO	LYETHYLENI	E/POLYPI	ROPYLENE	COMPOSITES	WITH ONE AU	XILIARY CLAM	
RUN	STRESS	% PP	% PE	$A_1(X 10^3)$	$A_{2}(X 10^{3})$	A ₃ (HR. ⁻¹)	A ₁ (X 10 ⁵)
distribution of the second	(PSI)			(IN.)	(IN.)		(IN./HR.)
1 2 3 4 5 6 7 8 9 10 11 12 13 14	1750.5			9.03 12.22 12.43 11.52 12.41 11.62 20.33 17.95 23.06 22.06 18.74 15.22 28.27 27.05	3.22 17.18 15.84 15.79 15.52 16.20 26.37 24.37 36.18 31.48 32.19 29.79 52.50 46.26	0.48 0.47 0.60 0.47 0.73 0.57 0.75 0.90 0.80 0.94 0.78 1.44 0.77 0.82	2.29 8.15 8.38 8.99 11.37 8.16 12.34 18.04 17.87 24.24 23.85 27.82 27.17 37.64
PO	LYETHYLENI	E/POLYPI	ROPYLENE	COMPOSITES	WITH TWO AU	XTLIARY CLAMI	es
RUN	STRESS (PSI)	% PP	% PE	A ₁ (X 10 ³) (IN.)	A ₂ (X 10 ³) (IN.)	A ₃ (HR1)	A (X 10 ⁵) (IN./HR.)
1 2 3 4 5	999.3 999.3 1001.0 1501.5 1496.8	51.0 50.8 50.7 53.2 51.2	49.0 49.2 49.3 46.8 48.8	12.45 14.32 11.25 19.37 18.37	14.25 16.91 17.30 29.46 35.43	0.71 0.98 0.46 0.98 0.87	12.40 12.95 8.65 21.65 26.28

P	OLYETHYLENE	/POLYPRO	OPYLENE	COMPOSITES	WITH THREE	AUXILIARY CI	JAMPS
RU	N STRESS	% PP	% PE	A ₂ (X 10 ³)	A ₀ (X 10 ³)	A_(HR1)	$A_{\lambda}(X 10^5)$
	(PSI)			(IN.)	(IN.)	3	(IN./HR.)
1	989.2	50.9	49.1	14.55	16.27	0.41	8.00
2	1000.2	50.9	49.1	11.07	15.91	0.78	12.61
3	1503.0	54.1	45.9	22.82	31.93	0.96	23.83
4	1502.7	50.7	49.3	19.34	35.10	0.94	26.10
5	1494.7	52.7	47.3	17.96	30.92	0.78	20.23

POL	YETHYL E NE,	/POLYPRO	OPYLENE	COMPOSITES	WITH FOUR AU	XILIARY CLAM	PS
RUN	STRESS	% PP	% PE	A ₃ (X 10 ³)	A ₂ (X 10 ³)	A ₂ (HR. ⁻¹)	A, (X 10 ⁵)
	(PSI)			(IN.)	(IN.)	3	(IN./HR.)
1	992.2	51.0	49.0	10.94	18.71	0.39	8.43
2	999.3	50.7	49.3	12.90	14.08	0.77	11.60
3	1494.7	52.7	47.3.	17.97	30.92	0.78	20.23
4	1502.0	51.8	48.2	19.21	32.87	0.91	23.19

	PE	/PP/PE	COMPOSI	res without	AUXILIARY CL	AMPS	
RUN	STRESS	% PP	% PE	A ₁ (X 10 ³)	A ₂ (X 10 ³)	A ₃ (HR. ⁻¹)	A _{),} (X 10 ⁵)
	(PSI)			(IN.)	[IN.)		(ÎN./HR.)
1 2 3 4 5 6 7 8 9 10	749.6 750.0 1001.0 1000.6 1250.5 1247.2 1503.7 1500.4 1751.1	34.1 34.0 34.0 33.9 34.0 33.8 33.6 33.9 33.9	65.9 66.0 66.0 66.1 66.2 66.4 66.1 66.1	8.27 8.85 14.81 9.66 15.33 17.64 21.87 22.64 28.37 27.33	8.68 10.75 17.94 15.90 31.85 28.38 39.34 42.29 57.21 64.58	1.19 0.61 1.09 1.06 0.70 0.97 0.93 0.76 0.85 0.67	8.04 6.09 11.87 13.76 12.93 19.25 27.18 18.85 46.28 33.56

	PP,	/PE/PP	COMPOSI	TES WITHOUT	AUXILIARY CL	AMPS	the entropy of the control of the co
RUN	STRESS	% PP	% PE	A ₃ (X 10 ³)	A (X 10 ³)	A_(HR1)	A (X 10 ⁵)
	(PSI)			(IN.)	² (IN.)	3	(IN./HR.)
1 2 3 4 5 6 7 8 9	751.5 755.2 1000.7 1001.5 1246.2 1250.8 1491.5 1502.7 1748.3 1753.3	67.2 67.3 67.4 67.8 67.3 67.3 66.8 67.7 67.5	32.8 32.7 32.6 32.2 32.7 32.7 33.2 32.3 32.5 33.0	8.75 7.54 11.64 10.72 13.62 16.14 16.50 18.38 24.88 21.58	5.95 7.40 10.65 12.49 17.52 19.23 25.29 26.59 36.60 35.51	0.90 0.52 0.74 0.60 1.14 1.20 1.45 0.85 1.13	5.72 4.33 7.63 6.35 16.40 14.84 22.56 17.81 25.65 33.17

-		SINGLE 1	SINGLE POLYETHYLENE SPECIMENS (TRANSVERSE ORIENTATION)					
STREET, S	RUN	STRESS	% PP	% PE	A ₁ (X 10 ³)	A ₂ (X 10 ³)	A ₂ (HR. ⁻¹)	A (X 10 ⁵)
de allanua		(PSI)			(IN.)	(IN.)		(IN./HR.)
	1 2 3 4 5	254.3 497.5 748.7 998.2 1251.1	0.0	100.0	2.19 5.13 9.20 12.16 16.83	2.35 6.57 14.19 22.64 36.45	1.48 1.45 1.45 1.08 1.00	2.18 5.07 10.07 14.31 21.18

APPENDIX B

FORTRAN 60 COMPUTER PROGRAMS USED IN ANALYSIS

Two basic Fortran 60 computer programs were used in calculating the parameters ${\bf A}_1$ through ${\bf A}_{l_4}$ and comparing the data with the least squares fitted equation:

$$\Delta L = A_1 + A_2(1 - \exp(-A_3t)) + A_Lt.$$

The first program called PROGRAM CREEP is a modification of a basic Least Squares program developed by Dr. W. Tolles of the Materials Science and Chemistry Department of the Naval Postgraduate School. The program is annotated by comment cards and needs no further explanation. The second program called PROGRAM CREEPDRW is a drawing program used to check the fitted equation with the data used. The off-line plotting subroutine J7-NPS-DRAW programmed by J. R. Ward of NPGS was available in the CDC 1604 Digital Computer System. Test data is included with each program for reference only and should not be used as actual data.

..JOB0630,HOWARD MA

PARAMETERS (YOU MAY WISH TO USE ONLY X) NOF IS THE FUNCTION NUMBER - USED FOR BRANCHING TO DIFFERENT PARTS OF THE EQN SUBPROGRAM WHEN SEVERAL FUNCTIONS ARE TO BE FIT (FOR SEVERAL JOBS TO BE DONE).

THE DIMENSIONED ARRAYS HAVE THE FOLLOWING MEANING--MINIMIZE THE SUM OF ERRORS SQUARED X, XB, AND XC ARE THE THREE INDEPENDENT WRITE FUNCTION SUBPROGRAM TO CALCULATE THE FUNCTION TO BE LEAST SQUARED. A IS DIMENSIONED 10 AND CONTAINS THE PARAMETERS TO BE VARIED SUCH AS TO PROGRAM CREEP

A(1) CONTAINS THE PARAMETERS WHICH ARE TO BE VARIED.

X(200), XB(200), AND XC(200) CONTAIN THE INDEPENDENT VARIABLES.

DEPENDENT Z(200) CONTAINS THE OBSERVED VALUES OF THE FUNCTION (THE OBSERVED

SO THAT THE PROGRAM MAY TAKE NUMERICAL DERIVATIVES WITH REASONABLE ACCURACY. FINCR(10) CONTAINS THE MAGNITUDE OF THE INCREMENTS FOR THE PARAMETERS A(10) R(200) CONTAINS THE DIFFERENCE BETWEEN OBSERVED (2(200)) AND CALCULATED E(10) CONTAINS THE ESTIMATES OF THE ERRORS OF THE PARAMETERS A(10) AFTER

I TERATION.

FIRST CARD - WILL BE REPRODUCED ON OUTPUT USED FOR LABELING. SECOND CARD FORMAT(12,13,12,13,1E10.2,55X,15) INPUT

IR NUMBER OF PARAMETERS TO BE VARIED

IS= NUMBER OF POINTS

NOF = FUNCTION NUMBER (SEE ABOVE)

NINP = NUMBER OF INDEPENDENT PARAMETERS

OF THE RESIDUAL CHANGES BY LESS THAN EPSIN IN TWO SUCCESSIVE ITERATIONS, EPSIL = IS USED AS A CRITERION FOR CONVERGENCE. IF THE RELATIVE VALUE

IO . SIGNAL FLAG WHEN NOT ZERO CAUSES COEFFICIENT MATRIX TO BE PRINTED. CONVERGENCE IS REACHED.

RUN = IDENTIFICATION CODE (6 ALPHANUMERIC FIGURES) THIRD CARD FORMATIA6,1F9.0,3F10.0)

WT = LOAD WEIGHT APPLIED TO LEVER ARM OF MACHINE.
APP = FRACTIONAL AREA OF SPECIMEN WHICH IS POLYPROPYLENE.
APE = SAME AS APP EXCEPT APPLIES TO POLYETHYLENE.

ARM = MACHINE LEVER ARM RATIO.

FOURTH CARD(S) - FORMAT (5E14.6) - CONTAIN YOUR ESTIMATE OF THE PARAMETERS,

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                                                                                                                                                                                     FUNCT ION
                                                                                                                                                                                                                                                                                                        (55H THE INCREMENTS FOR OBTAINING NUMERICAL DERIVATIVES ARE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     124 FORMAT (66H ESTIMATES OF THE ERROR IN EACH PARAMETER ARE (STANDARD
                                                                                                                                                                                                                                                                                                                                                                                                                                                                          THE SUM OF THE SQUAR
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 IR * NO. OF PARAMETERS, IS * NO. OF POINTS, A IS ARRAY OF PARAMETERS,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            SUBROUTINE LEAST(IR, IS, A, X, XB, XC, Z, FINCR, EPSIL, NOITR, RRQ, NOF, R, E,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     FIT ARE)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               122 FORMAT (53H OBSERVED MINUS CALCULATED VALUES OF THE BEST
                                                                                                                                                                                    NUMBER OF POINTS
                                                                                                                                                                                                                                                                                                                                                                                                                                (38H THE BEST VALUES OF THE PARAMETERS ARE)
                                                                                                                                                                                                                                                                                                                                                (34H THE OBSERVED VALUES TO BE FIT ARE)
                                                                                                                                                                                                                        FORMAT (8X,12,24X,13,17X,12,19X,12,12X,E10,2)
                                                                                                                                                                                                        EPSILON)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       (24H NUMBER OF ITERATIONS = ,12,50H
                                                                                                                                                                                                                                                                                                                                                                                         (30H THE INDEPENDENT VARIABLES ARE)
                                                                                                                                                                                                                                                                 (26H THE PARAMETERS FED IN ARE)
                                      (I2,I3,I2,I3,IE10,2,55X,I5)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         1ES OF THE ERRORS IS NOW = ,E12.6)
                                                                                                                                                                                107 FORMAT (98H NUMBER OF PARAMETERS
                                                                                                                                                                                                      NUMBER OF INDEP VAR
 (4X, A6, F11, 2, 2F12, 4)
                  (A6,1F9.0,3F10.0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                    (1H ,7E17,10)
                                                                                                                                                                                                                                                                                     (1H ,5E14.6)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    (1H •9E12•5)
                                                                                                                                                                                                                                                                                                                                                                                                              (1H ,5E14.6)
                                                                                                                                                                                                                                                                                                                                                                     (5E14.6)
                                                         (5E14.6)
                                                                            (14F5.1)
                                                                                                (7F10.5)
                                                                                                                                                                                                                                           109 FORMAT (1H)
                                                                                                                     (1HI)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           DEVIATION)
                                                                                                                                          (80H
                                                                                                                                                                                                     1 NUMBER
                                                                                                                                         106 FORMAT
                                                                                                                                                                                                                                                                                     FORMAT
                                                                                                                                                                                                                                                                                                         112 FORMAT
                                                                                                                                                                                                                                                                                                                                                                                                                               119 FORMAT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                        FORMAT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    FORMAT
                                                                                                                                                                                                                                                                 110 FORMAT
                 FORMAT
                                                                                                                                                                                                                                                                                                                                                 FORMAT
FORMAT
                                                                                                                                                                                                                                                                                                                                                                     114 FORMAT
                                                        FORMAT
                                                                                                                                                                                                                                                                                                                                                                                                                                                    120 FORMAT
                                     FORMAT
                                                                             FORMAT
                                                                                                FORMAT
                                                                                                                     FORMAT
                                                                                                                                                                                                                                                                                                                                                                                         FORMAT
                                                                                                                                                                                                                                                                                                                                                                                                              FORMAT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                STOP
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    END
                                                                                                                                                                                                                                                                                     1111
                                                                                                                                                                                                                                                                                                                                                 113
                                                                                                 103
                                                                                                                     105
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                200
                                                                            102
                                                        101
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    UU
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DIMENSION A(10), X(200), XB(200), XC(200), Z(200), FINCR(10), R(20
                                                                                                                10), D(200,10), DT(10,200), DEL(10), DS(10), DPI(10,10), DP(10,10),
PARAMETERS. EPSIL IS -FRACTIONAL- ERROR CRITERION. NOITR IS NO. OF
                      ITERATIONS REQUIRED (UP TO 10). RRQ = SUM OF SQUARES OF RESIDUALS.
                                                                  E IS THE ARRAY OF ESTIMATED ERRORS IN THE PARAMETERS.
                                           NOF IS NUMBER OF THE FUNCTION USEN IN EQN-.
                                                                                                                                                                                                         R(I) = Z(I) - EQN(A,X(I),XB(I),XC(I),NOF)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     CALL GAUSS3 (IR.1.00E-30.DP.DPI.KER)
                                                                                                                                                                                                                                                                                                                                                    D(I \rightarrow J) = EQN(A \rightarrow X(I) \rightarrow XB(I) \rightarrow XC(I) \rightarrow NOF)
                                                                                                                                                                                                                                                                                                                                                                                                                                           CONST = EQN(A,X(I),XB(I),XC(I),NOF)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         D(I,J) = (D(I,J)-CONST)/FINCR(J)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              DP(I,1) = DP(I,1)+DT(I,K)*D(K,1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               PRINT 102, (D(J,I), J=1,IS)
                                                                                                                                                                                                                                 IF (NOITR-9) 130,130,104
                                                                                                                                                                                                                                                                                                                                                                          A(J) = A(J)-FINCR(J)
                                                                                                                                                                                                                                                                                                     A(C) # A(C)+FINCR(C)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    FORMAT (1H ,11F10.3)
                                                                                                                                                                                                                                                     NOITR = NOITR + 1
                                                                                                                                                                                                                                                                             DO 220 J = 1, IR
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               IF (IO) 31,33,31
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               D(1)0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   1.IR
                                                                                                                                                                                                                                                                                                                          DO 15 I = 1,15
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  BO 30 J = 1,1R
                                                                                                                                                                                                                                                                                                                                                                                                                      DO 30 I = 1,15
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          1 . IR
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           1.IR
                                                                                                                                                                                   DO 20 I = 1,15
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    # 1,15
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         = 1,15
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      BO 32 I=1, IR
                                                                                                                                                              NOITR # 0
                                                                                                                                                                                                                                                                                                                                                                                                 GONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 (C.I) do
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   DO 35 I
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       DO 35 J
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               DT(J,1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        DO 36 K
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     DO 36
                                                                                                                                                                                                                                                                                                                                                                                                 220
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          30
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    102
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               35
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R(I) = Z(I) - EQN(A_0X(I)_0XB(I)_0XC(I)_0NOF
                                                                                                                                                                                                                                                                                           CRES = ABSF(RRQ-RRP) - EPSIL*RRP
                                                                                                                                                                                                                                                                                                                                                      FORMAT (20H CONVERGENCE FAILURE)
                                                                   DS(J) = DS(J)+DPI(J,K)+DT(K,I)
                                                                                                                                                                          DEL(I) = DEL(I) + DI(I + J) * R(J)
                                                                                                                                                                                                                                                                                                                                                                                                FORMAT (16H MATRIX SINGULAR)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       E(1) = SQRTF(RRQ/(SUM*FISI)
IF(KER-1) 120,37,120
                                                                                                                                                                                                                                                                                                                         IF(CRES) 100,100,25
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       SUM = SUM+D(J+I)**2
                                                                                                                                                                                                        A(I) = A(I) + DEL(I)
                                                                                                                                                                                                                                                                              RRG = RRG+R(I)**2
                                                                                                                                                           BO 110 J = 1, IS
                                                                                                                                                                                                                                                                                                                                                                                                                              DO 150 I = 1, IR
                                                                                                                                                                                                                                                                                                                                                                                                                                                          DO 140 J = 1,15
                                                                                                DT(L_01) = DS(L)
            BO 39 L # 101R
                                                                                                                                                                                         BO 10 I = 1,1R
                                                      BO 38 K = 1,1R
                                                                                                                                                                                                                     80320 I = 1,15
                                                                                                                                                                                                                                                                DO 50 I = 1,15
                                                                                                                             DO 110 I = 1,
                                                                                                                                             DEL(I) = 0.0
                                                                                                                                                                                                                                                                                                                                                                                                                FIS1 = 15-1
                                                                                                                                                                                                                                                                                                                                                                                   PRINT 1001
                                                                                                                                                                                                                                                 RRG = 0.0
                                                                                                                                                                                                                                                                                                          RRP = RRQ
                                                                                                                                                                                                                                                                                                                                        PRINT 101
                                                                                                                                                                                                                                                                                                                                                                    GO TO 100
                                                                                                                                                                                                                                                                                                                                                                                                                                            SUM # 0.0
                                                                                                               CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      RETURN
             37
                                                                                                                                                                          110
                                                                                                                                                                                                                                    320
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      150
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                                                                                                                                                                                                                                                                                                                                                                                                               100
                                                                                                                                                                                                                                                                                                                                                                                                                                                                        140
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                                                                                                                                                                                                                                                                                                                                        104
                                                                                                                                                                                                                                                                                                                                                                                               1001
                                                                                                                                                                                                                                                                                                                                                     101
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GAUSS3(N.EP.A.X.KER)
                       DIMENSION A(10,10), X(10,10)
                                                                                                                                                                                                                                                                                                                       IF (ABSF (A(L,L))-EP)50,50,30
                                                                                                                                                    IF (Z-ABSF (A(K,L)))11,12,12
                                                                                                                                                                                                                                                                                                                                                                                                               A(K.J) = A(K.J) - RATIO* A(L.J)
                                                                                                                                                                                                                                                                                                                                                                                                                                       X(K,J)=X(K,J)-RATIO*X(L,J)
                                                                                                                                                                                                                                                                                                                                                                                      RATIO=A(K+L)/A(L+L)
                                                                                                                                                                                                                                                                                                                                                                          IF (A(K,L))32,36,32
                                                                                                                                                                                                      IF (L-KP) 13,20,20
                                                                                                                                                                                                                                                                                                                                    IF (L-N) 31,34,34
                                                                                                                                                                                                                                                                                              X(L,J)=X(KP,J)
                                                                                                                                                                                                                                            A(L.) = A(KP.)
                                                                                                                                                                Z=ABSF(A(K,L))
                                                                                                                                                                                                                                                                                                                                                             BO 36 K=LP1.N
                                                                                                                                                                                                                                                                                                                                                                                                 BO 33 J=LP1.N
                                                                                                                                                                                                                                                                                                                                                                                                                             BO 35 J=1,N
                                                                                                                                                                                                                   BO 14 J=L .N
                                                                                                                                                                                                                                                                   DO 15 J=1.N
                                                                                                  DO 34 L=1.N
                                                                                                                                       DO 12 K=L.N
                                   DO 1 I=1,N
                                               DO 1 J=1.N
                                                             X(I . L) #0.0
          SUBROUTINE
                                                                        BO 2 K=1.N
                                                                                     X(K,K)=1.0
                                                                                                                                                                                                                                                                                                           X(KP,J)=Z
                                                                                                                                                                                                                                                         A(KP,))=2
                                                                                                                                                                                                                                                                                 Z=X(L,J)
                                                                                                                                                                                                                               Z=A(L,J)
                                                                                                                                                                                                                                                                                                                                                                                                                                                      CONTINUE
                                                                                                                                                                                         CONTINUE
                                                                                                                                                                                                                                                                                                                                                LP1=L+1
                                                                                                                           0.0=2
                                                                                                                                                                             KP=K
                                                                                                              KP=
                                                                                                  10
                                                                                                                                                                                                                                                                                                            15
                                                                                                                                                                                                                                                                                                                                   30
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0020

0021

0022 0023 0024 0025 0026 0027

0029

0030 0031 0032 0033

0015

0017

0004

0002

8000

7000

00100

0011

0012 0013 0014

62

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01
02
04
05
        0037
               0038
                       0039
                              0040
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                                                    0043
                                                            0044
                                                                    0045
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                                                                                           0048
                                                                                                   6400
                                      0041
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                                                                                                                                                                                                                                                  833
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                                                                                                                                                                                                                                                                 85
                                                                                                                                                                                                                                                           2
                                                                                                                                                                                                                                   192
                                                                                                                                                                                                                                   184
                                                                                                                                                                                                                                           470
                                                                                                                                                                                                                                                  969
                                                                                                                                                                                                                                                          4167
                                                                                                                                                                                                                    1.0E-05
                                                                                                                                                                                                                                                                   9
                                                                                                                                                                                                                           20.0E-05
                                                                                                                                                                                                                                   175
                                                                                                                                                                                                                                          452
                                                                                                                                                                                             PROGRAM CREEP
                                                                                                                                                                                                                                   162
                                                                                                                                                                                                                                          417
                                                                                                                                                                                                                                                  556
                                                                                                                                                                                                                                                          3333
                                                                                                                                                                                                                                                                         61333
                                                                                                                                              EGN=A(1)+A(2)*(1.-EXPF(-A(3)*X))+A(4)*X
                                                                                                                                                                                                            16.52
                                                                                                                                                                                                                   1.0E
                                                                                                                                                                                                                           1 • 0E
                                                                                                                                                                                                                                   150
                                                                                                                                                                                                                                          365
                                                                                                                                                                                                                                                                  26667
                                                                                                                                                                                                                                                         25
                                                                                                                                                                                                                                                                         48833
                                                                                                                                                                                                                                   145
                                                                                                                                                                                                                                          335
                                                                                                                                                                                             HOWARD, M.A.
                                                                                                                                                                                                            .131
                                                                   X(II,1))=(X(II,1)-S)/A(II,II)
                                                                                                                        FUNCTION EQN(A+X+XB+XC+NOF)
                                                                                                                                                                                                                                   142
                                                                                                                                                                                                                                          315
                                                                                                                                                                                                                                                  278
                                                                                                                                                                                                                                                          1667
                                                                                                                                                                                                                                                                 18333
                                                                                                                                                                                                                                                                        28833
                                                                                                                                                                                                                   1.0E-05
                                                                                                                                                                                                                          20.0E-03
                                                                                                                                       IF (A(3)*X-300.) 1,2,2
                                                                                                                                                              EGN=A(1)+A(2)+A(4) *X
                                                                                                                                                                                                                                  138
                                                                                                                                                                                                                                          285
                                                            S=S A(II +X(K+C)
                                    IF (II-N)41,43,43
                                                                                                                                DIMENSION A(10)
                                                                                                                                                                                            SAMPLE DATA
1 1.0E-4
                                                                                                                                                                                                                                                 139
                                                                                                                                                                                                                                  131
                                                                                                                                                                                                                                          272
                                                   80 42 K=IIP1,N
                                                                                                                                                                                                                                                          1388
                                                                                                                                                                                                                                                                        19667
       DO 43 I=1,N
                     BO 43 J=1.N
                                                                                                                                                                                                                                  128
                                                                                                                                                                                                                          E-03
                                            IIP1=II+1
                                                                                                                                                                                                           24.9
              I I = N+1-I
                                                                                                                                                                                                                   1. E-05
CONTINUE
                                                                                                                                                      GO TO 3
                                                                                                                                                                     RETURN
                                                                                  RETURN
                                                                                                 RETURN
                             S=0.0
                                                                                          KER=2
                                                                           KER=1
                                                                                                                                                                                                                                         228
                                                                                                                                                                                                                                                                1999
                                                                                                                                                                                                                                                         1111
                                                                                                          END
                                                                                                                                                                             END
                                                                                                                                                                                    END
                                                                                                                                                                                                   4 28 1
                                                                                                                                                                                            TEST
                                                                                          50
                                                                                                                                                                                                                                  110
                                                                   43
                                            41
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CONST SERVES ONLY TO SIMPLIFY
                                                                              IMAX IS MAX VALUE OF TIME, IS AND IT ARE NO. OF POINTS IN RESPECTIVE DATA.
                                                                                                                                   PX AND QX ARE TIME
                                                                                                                                                                                                              ITITLE AND OTHER ARGUMENTS OF DRAW SUBROUTINE ARE FOUND IN J7-NPS-DRAW.
                                                    PROGRAM IS USED TO COMPARE TWO SETS OF DATA WITH THE FITTED EQUATION.
                                                                                                        A AND B ARE PARAMETERS FOUND FOR TWO SETS OF DATA IN PROGRAM CREEP.
                                                                                                                                                                                                                                                                    19X(90), P(30), Q(30), U(30), V(30), YA(200), YB(200), T(200)
                                                                                                                                                                                                                                        DIMENSION A(4), B(4), ITITLE(12), PT(90), QT(90), PX(90),
                                                                                                                                  PT AND GT ARE ELONGATION DATA FOR TWO SETS OF DATA.
                                                                                                                                                        DATA CORRESPONDING TO PT AND GT RESPECTIVELY.
                                                                                                                                                                                  INPUT AND FOR TEST DATA EQUALS 1000.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     FORMAT (1F5.0,215,1F10.0)
                                                                                                                                                                                                                                                                                               READ 21, TMAX, IS, IT, CONST
                                                                                                                                                                                                                                                                                                                                                                               READ 23, (PT(I), I=1, IS)
                                                                                                                                                                                                                                                                                                                                                                                                       READ 24, (PX(I), I=1, IS)
                                                                                                                                                                                                                                                                                                                                                                                                                                 23, (QT(1), I=1, IT)
                                                                                                                                                                                                                                                                                                                                                                                                                                                           READ 24, (QX(I), I=1,IT)
                                                                                                                                                                                                                                                                                                                        READ 22, (A(I), I=1,4)
                                                                                                                                                                                                                                                                                                                                                   22, (B(I), I=1,4)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       @T(1)=QT(1)/CONST
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          PT(1) = PT(1) / CONST
                          PROGRAM CREEPDRW
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         FORMAT (4E14.3)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               FORMAT (4E14.2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           FORMAT (14F5.1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     FORMAT (7F10.5)
.. JOB0630, HOWARD MA
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     I(I)=I(I-1)+R
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   PRINT 26.A.B
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          DO 3 1=2,200
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  DO 2 I=1,IT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             DO 1 1=1,15
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      R=TMAX/200.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               FORMAT(13)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                T(1)=0.0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 CONTINUE
                                                                                                                                                                                                                                                                                                                                                   READ
                                                                                                                                                                                                                                                                                                                                                                                                                                 READ
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               22
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     77
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CALL DRAW(200.YA,T,1,0,LABEL,ITITLE,0,,0,0,0,0,0,0,8,15,1,LAST)
                                                                                                                                                                                                                                                                                                                                                                                                                                                     ITITLE(7)=8H CREEP 0
                                                                                                                                                                                                                                                                                                                                                                                                                          ITITLE(1)=8H HOWARD
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   ITITLE(8)=8HF PE/PP
           YA(I)=EQN(A+T(I))
                                                                                                                                                                                                                           IF (30-1T) 11,13,13
                                                      YB(I)=EQN(B,T(I))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                ITITLE(9)=8H SIX
                                                                                                                                                                                                                                                                                                                                                                                                                                      ITITLE(2) =8HMA
                                                                                IF (30-IS) 6,8,8
                                                                                                            P(I)=PT(2*1-1)
                                                                                                                          W(I)=PX(2*I-1)
                                                                                                                                                                                                                                                     0(1) #QT(2*1-1)
                                                                                                                                                                                                                                                                    V(I) = QX(2*I-1)
                                                                                                                                                                                                                                        BO 12 I=1,30
BO 4 I=1,200
                                                                                                                                                                                                                                                                                                                                                                                                             LABEL= 4H S=
                                       80 5 I=1,200
                                                                                                                                                                                                                                                                                                                                                                    DO 16 I=1,12
                                                                                                                                                                                                                                                                                                                                                                                 ITITLE(I)=8H
                                                                                               DO 7 I=1,30
                                                                                                                                                                    BO 9 I=1,15
                                                                                                                                                                                                                                                                                                             DO 14 I=1 1
                                                                                                                                                                                 P(I) = PI(I)
                                                                                                                                                                                               U(I)=PX(I)
                                                                                                                                                                                                                                                                                                                          (I)=01(I)
                                                                                                                                                                                                                                                                                                                                        V(I)=0X(I)
                         CONTINUE
                                                                   CONTINUE
                                                                                                                                                       GO TO 10
                                                                                                                                                                                                                                                                                                                                                      CONTINUE
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U

APPENDIX C

SPECIAL EQUIPMENT

Viscoelastic studies of materials require many sets of data with long time scale measurements. This necessitates the availability of many independently operated measuring devices. This requirement led to the design and construction of additional Creep machines operated by a simple lever arm device for loading and a series of ball and socket swivels with turnbuckles for varying the length and size of specimens used. Figures 39 through 42 are pictures and drawings of the constant load Creep machines designed and built in this project.

Additionally, since stress relaxation measurements are necessary in characterizing the viscoelastic properties of materials, a multiple capacity stress relaxation (constant strain) machine was designed. This machine is still in the design and construction stages. Figure 43 is a drawing of the machine.

A necessary item in the measurement of creep and relaxation data is the measuring device and recording equipment. The potential advantages of automatic recording of electrical signals made it desirable to use conventional wire strain gauges. However, their very limited extension capability and the requirement of attaching them to the specimen made it necessary to fabricate a special fixture to increase their extension capability. Figure 44 is a schematic of the clip-on electromechanical extensometer devised. It consists of a phosphor bronze blank, bent to the dimensions seen, with two BLH SR-4 type A-5-1 strain gauges attached as shown in Figure 45. The use of a simple aluminum frame and rubber bands serve to clip the extensometer to the specimen as shown in Figures 46 and 47. The attached strain gauges serve as one-

half of a Wheatstone bridge and when connected as seen in Figure 48, can be calibrated to give a signal proportional to the strain in the specimen. The relationship between the strain at the center of the face of the clip gauge and the elongation between the feet of the clip gauge is seen in the following analysis.

Figure 49 is a schematic drawing of the blank used in the clip gauge. Marin and Sauer [12] show that the total axial strain on the blank is:

(1)
$$\delta = \begin{bmatrix} \frac{Hh}{FI} \end{bmatrix} \begin{bmatrix} \frac{2}{3} + \frac{L}{h} + \begin{pmatrix} \frac{L}{h^3} \end{pmatrix} \frac{I}{A} \end{bmatrix}$$

where A = cross-sectional area of the blank (bd)

I = moment of inertia of the area $(bd^3/12)$

E = modulus of elasticity in the blank.

The unit strains at point P (where the SR-4 strain gauges are centered)
are:

(2)
$$\mathcal{E}_1 = -\mathcal{E}_2 = \frac{1}{E} \left[\frac{H}{A} \pm \frac{6M}{bd^2} \right]$$
 or $\frac{H}{Ebd} \left(1 \pm \frac{h}{d} \right)$

Placing the value of H from Equation 1 into Equation 2, gives the following relation:

relation:
(3)
$$\mathcal{E}_{1} = -\mathcal{E}_{2} = \left(\frac{\delta}{h}\right) \frac{\left(\frac{d}{h}\right)^{2} \left(1 \pm \frac{d}{h}\right)}{8 + 12\frac{h}{h} + \left(\frac{h}{h}\right)\frac{h}{h}^{2}}$$

If the strain gauges are placed in series as in the Wheatstone bridge, the measured strain is given by:

(4)
$$\varepsilon = \varepsilon_1 - \varepsilon_2 = 2|\varepsilon_1| \propto \delta$$

The completed clip type extensometer was tested for linearity and hysterisis on a device similar to Figure 50. The calibration curve is shown in Figure 51. A modification of PROGRAM CREEP given in Appendix B will readily convert the mv output of the Wheatstone bridge into strain in inches per inch. A simple block diagram of the complete recording

system for Creep data is given in Figure 52. A similar diagram for the control and recording system of the Stress Relaxation machine is given in Figure 53.



FIC. 39 GREEP LABURATURY

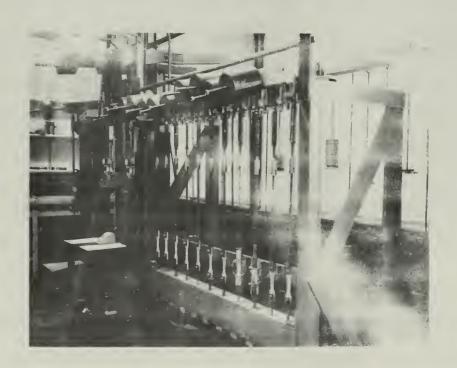


FIGURE 40 ELEVEN INIT GREEF MALE

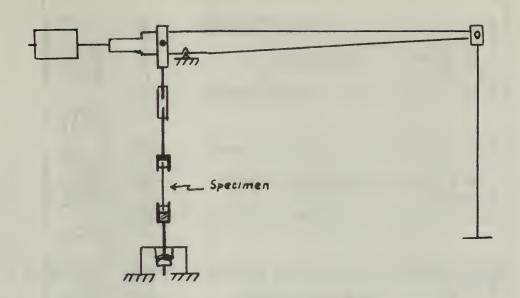


FIGURE 41 LEVER ARM ASSEMBLY

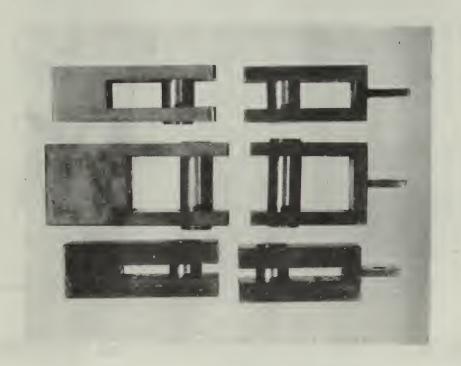


FIGURE 42 VARIOUS SPECIMEN HOLDERS

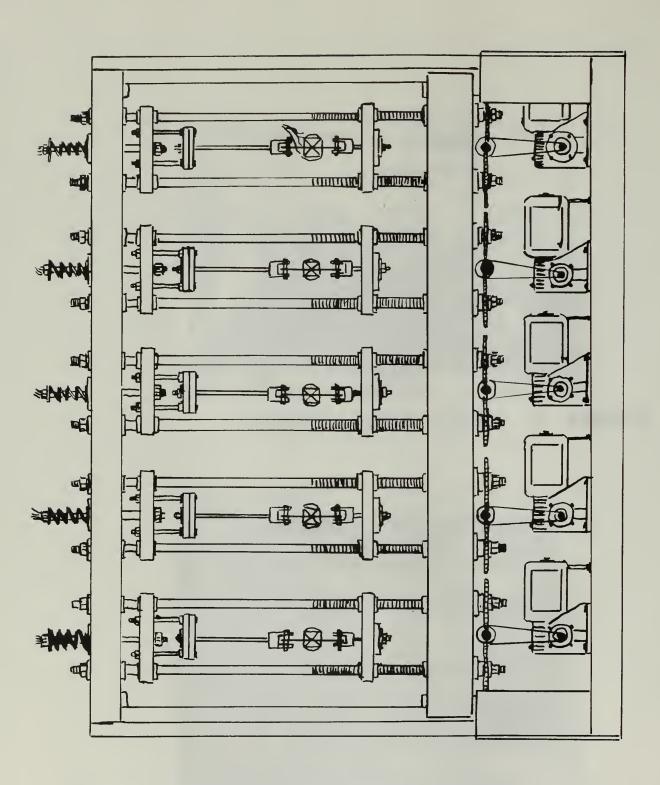


FIGURE 43 FIVE UNIT STRESS RELAXATION MACHINE

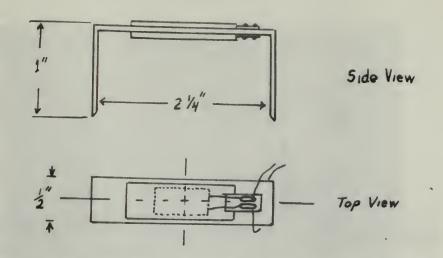


FIGURE 44 CLIP GAUGE EXTENSOMETER (DIMENSIONS)

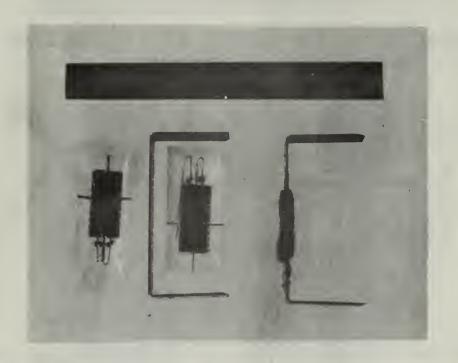


FIGURE 45 CLIP GAUGE EXTENSOMETER (COMPONENTS)

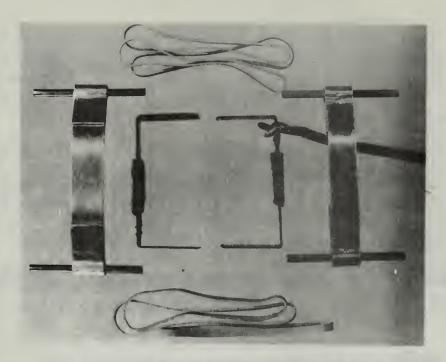


FIGURE 46 CLIP GAUGE EXTENSOMETERS & BRACKETS

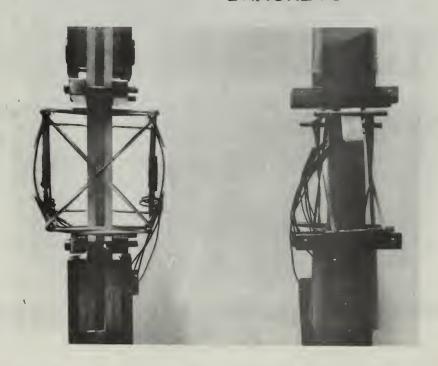


FIGURE 47 CLIP GAUGE EXTENSOMETER ATTACHED TO SPECIMEN

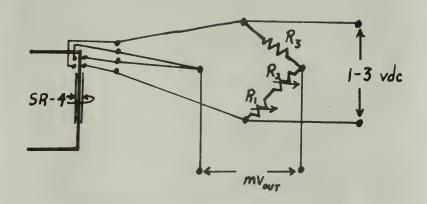


FIGURE 48 CLIP GAUGE EXTENSOMETER WHEATSTONE BRIDGE CIRCUITRY

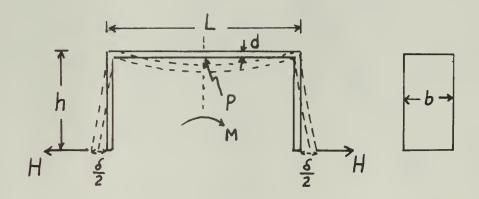


FIGURE 49 CLIP GAUGE EXTENSOMETER
THEORETICAL ANALYSIS SCHEMATIC

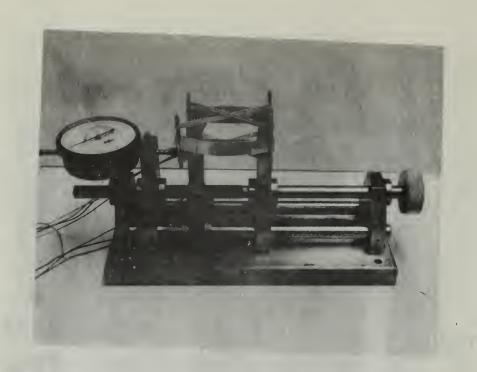


FIGURE 50 EXTENSOMETER CALIBRATION DEVICE

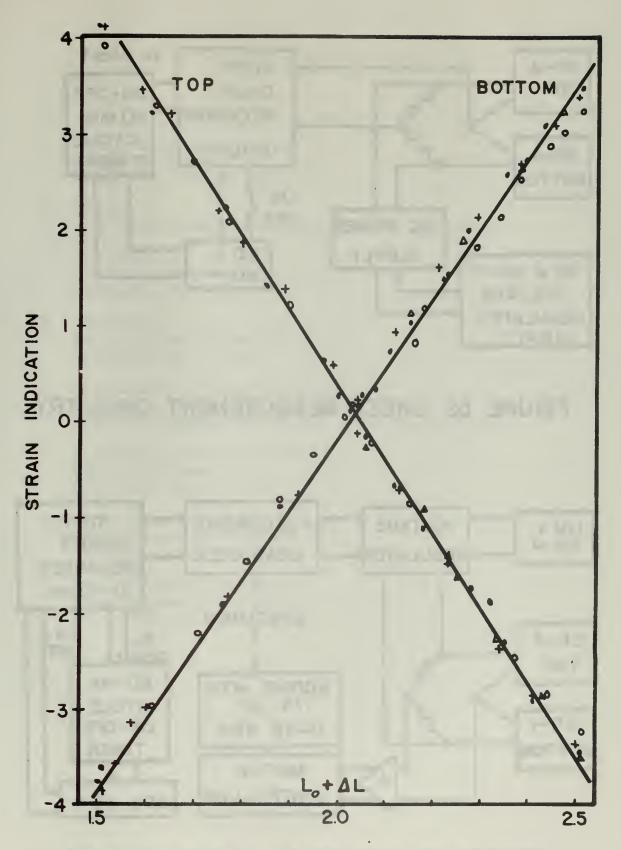


FIGURE 51 CLIP GAUGE EXTENSOMETER CALIBRATION CURVE

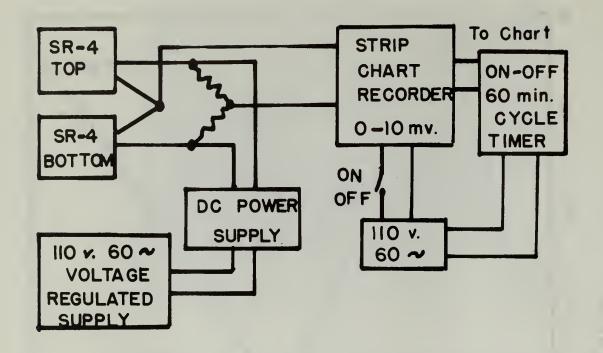


FIGURE 52 CREEP MEASUREMENT CIRCUITRY

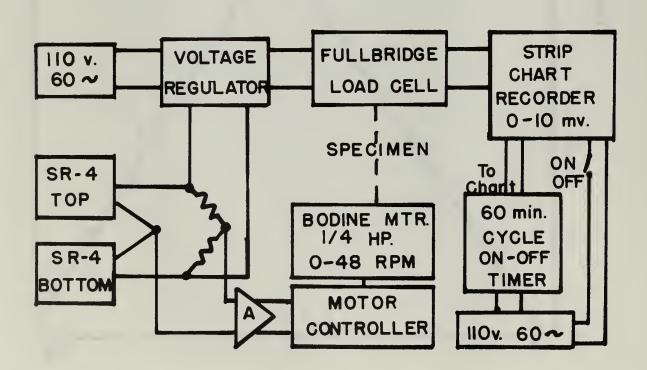


FIGURE 53 RELAXATION MEASUREMENT CIRCUITRY

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13. ABSTRACT

An investigation of uniaxial creep in simple synthetic composites of polyethylene and polypropylene was made to determine the parametric behavior and interrelation of each component by varying the relative volume and interfacial contact area. A mathematical model was developed and used to predict the experimental behavior which was determined by least squares fitting of the data. A digital computer was used in the analysis and good correlation between the experimental measurements and theoretical predictions was obtained. Included is a report of the design and construction of equipment for theoretically meaningful viscoelastic measurements.

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14.	KEY WORDS	LI	LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	wT	
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CREEP								
PLASTIC								
POIYETHYLENE								
POLYPROPYLENE								
LAMINATED				-				
POLYMER								
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